As promised, we tackle some 700+ level questions today. Mind you, these questions are not your typical GMAT type questions. The reason we are discussing them is that they look mind boggling but are easily workable when the concepts of relative speed are used. They give insights that help you understand relative speed. Once you are good with the concepts, you can solve most of the relative speed based questions easily.

Question 1: Cities A and B are 20 miles apart. From both of these cities, simultaneously, two people start walking toward each other at a constant speed of 2 miles/hr. At the same time, a dog leaves city B and runs at a constant speed of 5 miles/hr toward city A. When it reaches the person from city A, it immediately turns around and runs back to the person from city B. When it reaches the person from city B, it turns around and runs back to the person from city A. It keeps doing so until the two people meet. How many miles did the dog run?

(A) 15 miles
(B) 20 miles
(C) 25 miles
(D) 30 miles
(E) 35 miles

Solution: Looks really tricky, doesn't it? Actually it is really easy once you look at it from the right perspective. It would take you less than a minute to solve almost any 700+ level GMAT question if you can figure out the most optimum method to solve it. The point is – how long will it take you to figure out the most optimum method? Take a minute to think what you would do in this question.

We know the dog's speed. We need to know the distance it has run. Directly calculating distance is a little complicated since we need to consider the distance covered by the two men too. Instead, if we can figure out the time for which the dog ran, we can easily calculate the distance. For how long did the dog run? He started when the two men started from their respective cities and stopped when the two men met. Therefore, if we can find the time taken by the men to cover the 20 miles, we will get the time for which the dog ran.

This is where the relative speed concept comes in handy.

Total distance between the two men = 20 miles

Relative speed of the men with respect to each other = 2 + 2 mph (since they are travelling in opposite directions)

Time taken by the men to meet = 20/4 = 5 hrs

Therefore, the dog also ran for 5 hrs.

In 5 hrs, the dog must have run 5*5 = 25 miles

Answer (C)

I hope you see that the right perspective simplifies the question immensely. Let's look at another 700+ level question.

Question 2: A man cycling along the road at a constant speed noticed that every 12 minutes a bus overtakes him and every 6 minutes he meets an oncoming bus. If all buses move at the same constant speed and leave the bus station at fixed time intervals, what is the time interval between consecutive buses?

(A) 5 minutes
(B) 6 minutes
(C) 8 minutes
(D) 9 minutes
(E) 10 minutes

Solution: Again, the question is a little tricky but once you understand how to tackle it, it takes less than a minute.

Buses are coming from opposite directions. A bus overtakes the man every 12 minutes i.e. a bus moving in the same direction as the man overtakes him every 12 mins. To clarify: say, at constant intervals, buses leave a bus station located from where the man left and travel on the same road as the man. Since they are faster, they overtake the man. The man noticed that a bus overtakes him every 12 mins. Obviously then, they must be leaving at constant intervals. Also, he meets a bus coming from the opposite direction every 6 mins. So buses must be leaving from a bus station located at the opposite end of the road at constant intervals.

I hope the problem is clear to you. Now let's try to work out the solution.

Say the cyclist is stationary at a point. Buses are coming from opposite directions (same speed, same time interval). A bus will meet the cyclist every t minutes from either direction. This 't minutes' must be the time interval between consecutive buses. Let's say, a bus from each direction just met him. After t minutes, 2 more buses from opposite directions will meet him and so on… We need to find 't'.

Now the cyclist starts moving at speed c. His speed relative to the bus going in the same direction becomes b – c. His speed relative to the bus from the opposite direction becomes b + c. This is the reason that the time interval between two buses is different for the opposite directions. Time interval is in the ratio 12:6. Then, the ratio of the relative speed in the two cases must be inverse i.e. 6:12

(b-c):(b+c) = 6:12 which gives you c = (1/3)b

This means that the bus travelling at a relative speed which is 2/3rd of its usual speed (b-c = 2b/3) takes 12 minutes to meet the man. If it were travelling at its usual speed, it would have taken 12*(2/3) = 8 mins to meet the man. This 8 mins is the value of ‘t’ i.e. the time interval between buses.

Answer (C)

You might need to go through the question a few times before you fully understand it. It will also be helpful to draw a diagram and see what the situation looks like.